Period 3, Feb 12, 2025

BASIC INTEGRATION RULES ($a > 0$) Pg. 385

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u\,du = e^u + C$
7. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
8. $\int \sin u\,du = -\cos u + C$
9. $\int \cos u\,du = \sin u + C$
10. $\int \tan u\,du = -\ln|\cos u| + C$
11. $\int \cot u\,du = \ln|\sin u| + C$
12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
13. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u\,du = \tan u + C$
15. $\int \csc^2 u\,du = -\cot u + C$
16. $\int \sec u \tan u\,du = \sec u + C$
17. $\int \csc u \cot u\,du = -\csc u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

d) $\int \frac{1}{x\sqrt{4x^2 - 36}}dx$

$u = 2x \Rightarrow \frac{u}{2} = x$

$u^2 = 4x^2$

$du = 2dx$

$\frac{du}{2} = dx$

$\int \frac{1}{\frac{u}{2}\sqrt{u^2 - 6^2}} \cdot \frac{du}{2} = \int \frac{du}{\frac{u}{\cancel{2}}\cancel{2}\sqrt{u^2-6^2}} = \int \frac{du}{u\sqrt{u^2-6^2}}$

$\frac{1}{6}\operatorname{arcsec}\frac{|2x|}{6} + C$

5. $\boxed{\ln|a| - 2\ln|b| = \ln|a| - \ln b^2 = \ln\left|\frac{a}{b^2}\right|}$

Evaluate each integral.

a) $\int (\tan x + 2\sec x)\,dx = \int \tan x\,dx + \int 2\sec x\,dx$

$-\ln|\cos x| + 2\ln|\sec x + \tan x| + C$

$\ln\left|\frac{(\sec x + \tan x)^2}{\cos x}\right| + C$

3.

Evaluate each of the following.

a) $\frac{d}{dx}\left[\int_x^2 \sec t\, dt\right]$

$0 - \sec x$

b) $\frac{d}{dx}\left[\int_{x^2}^{3x} (e^{-t^2+1} + \sqrt{t})dt\right]$

$T = 3x$
$dT = 3dx$

$T = x^2$
$dT = 2x\,dx$

$\left(e^{-(3x)^2+1} + \sqrt{3x}\right) \cdot 3 - \left(e^{-(x^2)^2+1} + \sqrt{x^2}\right) 2x$

b) $\int \tan^2 x \sec^2 x\, dx$

$U = \tan x$
$dU = \sec^2 x\, dx$
$\frac{dU}{\sec^2 x} = dx$

$\int U^2 \cdot \cancel{\sec^2 x} \cdot \frac{dU}{\cancel{\sec^2 x}} = \int U^2 dU = \frac{1}{3} U^{2+1=3} + C$

$\frac{1}{3}(\tan x)^3 + C = \frac{\tan^3 x}{3} + C$

c) $\int \frac{e^x}{4 + e^{2x}} dx$

$U = e^x$
$U^2 = e^x \cdot e^x = e^{2x}$
$dU = e^x dx$
$\frac{dU}{e^x} = dx$

$\int \frac{x}{4+U^2} \cdot \frac{dU}{U}$

$\int \frac{\cancel{e^x}}{4+U^2} \cdot \frac{dU}{\cancel{e^x}} = \int \frac{dU}{2^2 + U^2} = \frac{1}{2} \arctan \frac{e^x}{2} + C$

19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

b) $\int \frac{4}{25x^2 + 9} dx = \int \frac{4}{(U^2 + 3^2)} \cdot \frac{dU}{5} = \frac{4}{5} \int \frac{dU}{U^2 + 3^2} = \frac{4}{5} \cdot \frac{1}{3} \arctan \frac{5x}{3} + C$

$U = 5x$
$U^2 = 25x^2$
$dU = 5dx \Rightarrow \frac{dU}{5} = dx$

19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

$\frac{4}{15} \arctan \frac{5x}{3} + C$

Evaluate each of the following.

a) $\int \frac{1}{\sqrt{9-x^2}}dx = \int \frac{1}{\sqrt{3^2 - x^2}} dx = \arcsin \frac{x}{3} + C$

18. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$

4.

a) $\int 5e^{5x}\,dx = \int \cancel{5} e^{u} \cdot \frac{du}{\cancel{5}} = \int e^u du = e^u + C = e^{5x} + C$

$u = 5x$

$du = 5dx$

$\frac{du}{5} = dx$

$$\int \frac{x-3}{\sqrt{9-x^2}}dx = \int \frac{x}{\sqrt{9-x^2}}dx - \int \frac{3}{\sqrt{9-x^2}}dx$$

$u = 9-x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$\int \frac{\cancel{x}}{\sqrt{u}} \cdot \frac{du}{-2\cancel{x}}$

$-\frac{1}{2}\int u^{-\frac{1}{2}} du$

$-\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{1} u^{\frac{1}{2}}$ $\quad -\frac{1}{2}+1=\frac{1}{2}$

$3\int \frac{dx}{\sqrt{3^2-x^2}}$

18. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$

$3\arcsin \frac{x}{3} + C$

$-\sqrt{9-x^2} - 3\arcsin \frac{x}{3} + C$

$$\int \frac{dx}{x^2+6x+25}$$

Complete The Square ax^2+bx+c

$a=1$

$b=6$

$\frac{b}{2}=\frac{6}{2}=3$

$\left(\frac{b}{2}\right)^2=(3)^2=9$

$$\underbrace{x^2+6x+9}-\underbrace{9+25}$$

$$(x+3)^2+16$$

$$\int \frac{dx}{(x+3)^2+16}$$

$u=x+3$

$du=dx$

$$\int \frac{du}{u^2+4^2}=$$

19. $\int \frac{du}{a^2+u^2}=\frac{1}{a}\arctan\frac{u}{a}+C$

$$\frac{1}{4}\arctan\frac{u}{4}+C$$

$$\frac{1}{4}\arctan\frac{x+3}{4}+C$$

$$\int \frac{x+7}{x^2+6x+25}\,dx$$

$$\int \frac{x+3+4}{x^2+6x+25}\,dx$$

$u=x^2+6x+25$

$du=(2x+6)\,dx$

$\frac{du}{2x+6}=dx$

$\frac{du}{2(x+3)}=dx$

$$\int \frac{x+3}{x^2+6x+25}\,dx+\int \frac{4}{x^2+6x+25}\,dx$$

$$\int \frac{\cancel{(x+3)}}{u}\cdot\frac{du}{2\cancel{(x+3)}}+\int \frac{4}{(x+3)^2+16}\,dx$$

$\frac{1}{2}\int \frac{du}{u}$

$\frac{1}{2}\ln|u|$

$\frac{1}{2}\ln|x^2+6x+25|$

$u=x+3$

$du=dx$

$+4\int \frac{du}{u^2+4^2}$

19. $\int \frac{du}{a^2+u^2}=\frac{1}{a}\arctan\frac{u}{a}+C$

$$\frac{1}{2}\ln|x^2+6x+25|+\cancel{4}\cdot\frac{1}{\cancel{4}}\arctan\frac{x+3}{4}+C$$

$$\ln\left|\sqrt{x^2+6x+25}\right|+\arctan\frac{x+3}{4}+C$$

$$\int x\sqrt{2x-1}\,dx = \int x\sqrt{u}\cdot\frac{du}{2} = \int \frac{(u+1)}{2}\cdot\frac{\sqrt{u}}{2}\cdot du$$

$u = 2x-1$

$du = 2dx$

$\frac{du}{2} = dx$

$u = 2x-1$

$u+1 = 2x$

$\frac{u+1}{2} = x$

$$\frac{1}{4}\int (u+1)u^{\frac{1}{2}}\,du$$

$$\frac{1}{4}\int \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du$$

$$\frac{1}{4}\left[\frac{2}{5}u^{\frac{3}{2}+1=\frac{5}{2}} + \frac{2}{3}u^{\frac{1}{2}+1=\frac{3}{2}}\right] + C$$

$\frac{5}{2} = 2\frac{1}{2}$

$\frac{3}{2} = 1\frac{1}{2}$

$$\frac{1}{4}\left[\frac{2}{5}(2x-1)^{5/2} + \frac{2}{3}(2x-1)^{3/2}\right] + C$$

$$\frac{1}{4}\left[\frac{2}{5}(2x-1)^{2}\sqrt{2x-1} + \frac{2}{3}(2x-1)^{1}\sqrt{2x-1}\right) + C$$

$$\frac{1}{4}\cdot 2\cdot (2x-1)^{1}\sqrt{2x-1}\left[\frac{1}{5}(2x-1) + \frac{1}{3}\right] + C$$

$$\int \frac{2x}{(x+1)^2}\,dx = \int \frac{2(u-1)}{u^2}\,du = 2\int \frac{u-1}{u^2}\,du$$

$u = x+1$

$u-1 = x$

$du = dx$

$$2\left[\int \frac{u}{u^2}du - \int \frac{1}{u^2}du\right]$$

$$2\left[\int \frac{1}{u}du - \int u^{-2}du\right]$$

$$2\left[\ln|u| - \frac{1}{-1}u^{-2+1=-1}\right] + C$$

$$2\left[\ln|x+1| + \frac{1}{x+1}\right] + C$$

$$\int \frac{x^2+x+1}{x^2+1}\,dx$$

$$x^2+1 \overline{\big)\, x^2+x+1}$$
$1 \quad R = x$
$-(x^2-0x+1)$
$0+x+0$

$$\int \left(1+\frac{x}{x^2+1}\right)dx$$

$$\int 1\,dx+\int \frac{x}{x^2+1}\,dx$$

$u=x^2+1$
$du=2x\,dx$
$\frac{du}{2x}=dx$

x

$$x+\int \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}}$$

$$x+\frac{1}{2}\int\frac{du}{u}= x+\frac{1}{2}\ln|u|+c = x+\frac{1}{2}\ln|x^2+1|+c = x+\ln\sqrt{x^2+1}+c$$

$4x-2 \quad R = -2x+5$

$$x^2+2x+1 \overline{\big)\, 4x^3+6x^2-2x+3}$$
$-(4x^3+8x^2+4x)$
$0x^3-2x^2-6x+3$
$-(-2x^2-4x-2)$
$0x^2-2x+5$

$$= 4x-2+\frac{-2x+5}{4x^3+6x^2-2x+3}$$

$$\int_0^2 x\sqrt{4-x^2}\,dx$$

x bounds

$u = 4-x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$\int \cancel{x}\sqrt{u}\cdot\frac{du}{-2\cancel{x}}$

$-\frac{1}{2}\int u^{\frac{1}{2}}du$

$\cancel{-\frac{1}{2}}\cdot\cancel{\frac{2}{3}}\cdot u^{\frac{1}{2}+1=\frac{3}{2}}$

$-\frac{1}{3}(4-x^2)^{\frac{3}{2}}\Big[_0^2$

$-\frac{1}{3}(4-2^2)^{3/2}-\left[-\frac{1}{3}(4-0^2)^{3/2}\right]$

$0 + \frac{1}{3}\cdot 8 = \frac{8}{3}$

Same

$$\int_0^2 x\sqrt{4-x^2}\,dx$$

$4-x^2=u$

when $x=2$

$4-2^2=0=u$

when $x=0$

$4-0^2=u$

$4=u$

u bounds

$u=4-x^2$

$du=-2x\,dx$

$\frac{du}{-2x}=dx$

$\int \cancel{x}\sqrt{u}\cdot\frac{du}{-2\cancel{x}}$

$-\frac{1}{2}\int_4^0 u^{\frac{1}{2}}du = \cancel{-\frac{1}{2}}\cdot\cancel{\frac{2}{3}}\,u^{\frac{3}{2}}\Big[_4^0$

$-\frac{1}{3}(0)^{3/2}-\left[-\frac{1}{3}(4)^{3/2}\right]$

$0+\frac{1}{3}(8)$

$\frac{8}{3}$

$$\int_0^{\frac{\pi}{4}} \tan x\ \sec^2 x\ dx$$

$\tan x = u$

$\sec^2 x\,dx = du$

$dx = \frac{du}{\sec^2 x}$

$\tan\frac{\pi}{4}=1$

$\tan 0 = 0$

$\int_0^1 u\,\cancel{\sec^2 x}\cdot\frac{du}{\cancel{\sec^2 x}}$

$\int_0^1 u\,du = \frac{1}{2}u^2\Big[_0^1 = \frac{1}{2}(1)^2-\frac{1}{2}(0)^2=\frac{1}{2}$

$$\int_0^{\frac{\pi}{4}} \tan x \ \sec^2 x \, dx$$

$u = \sec x$

$du = \sec x \tan x \, dx$

$du = u \cdot \tan x \, dx$

$\frac{du}{u \tan x} = dx$

$\sec \frac{\pi}{4} = \sqrt{2}$

$\sec 0 = 1$

$$\int_1^{\sqrt{2}} \cancel{\tan x} \cdot u^2 \cdot \frac{du}{\cancel{u \tan x}}$$

$$\int_1^{\sqrt{2}} u\,du = \frac{1}{2}u^2 + C \Big]_1^{\sqrt{2}}$$

$$\frac{1}{2}(\sqrt{2})^2 - \frac{1}{2}(1)^2$$

$$1 - \frac{1}{2} = \frac{1}{2}$$

$$\int_{-1}^{1} 3x^2 \sqrt{x^3+1} \, dx$$

$u = x^3 + 1 \Rightarrow du = 3x^2 dx \Rightarrow \frac{du}{3x^2} = dx$

$2 = 1^3 + 1$

$u = 2$ when $x = 1$

$(-1)^3 + 1 = -1 + 1 = 0$

$u = 0$ when $x = -1$

$$\int_0^2 \cancel{3x^2} \sqrt{u} \cdot \frac{du}{\cancel{3x^2}}$$

$$\int_0^2 u^{\frac{1}{2}} du = \frac{2}{3} u^{\frac{1}{2}+1=\frac{3}{2}} = \frac{2}{3} u^{3/2} \Big]_0^2 = \frac{2}{3}(2)^{3/2} - \cancel{\frac{2}{3}(0)^{3/2}}$$

$$\frac{2 \cdot 2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}$$

$$\int_0^3 \frac{x}{x^2+1}dx$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

when $x=0$

$u = 0^2+1$

$u = 1$

when $x=3$

$u = 3^2+1$

$9+1$

$u = 10$

$\ln 1 = 0$

$$\int_1^{10} \frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}} = \frac{1}{2}\int_1^{10}\frac{du}{u} = \frac{1}{2}\ln|u| + c \Big[_1^{10} = \frac{1}{2}\cdot\ln 10 - \frac{1}{2}\cdot\ln 1 = \frac{1}{2}\ln 10 - 0$$

$\ln\sqrt{10} \approx 1.151$

$$\int \frac{dx}{\sqrt{x^2-1}} \quad (Can't\ do.)$$

$$\int \frac{dx}{\sqrt{-1(1-x^2)}}$$

$$\int \frac{dx}{i\sqrt{1-x^2}}$$

can't work

18. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$